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MAP ACCURACY EVALUATION

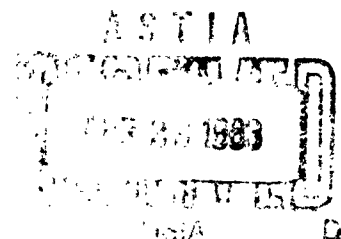
PART II

Evaluation of Vertical Map Information

JANUARY 1963



Aeronautical Chart and Information Center
United States Air Force
St. Louis 18, Mo.



MAP ACCURACY EVALUATION

Part II

Evaluation of Vertical Map Information

January 1963

Prepared by
Geo-Sciences Branch
Chart Research Division

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Aeronautical Chart and Information Center
United States Air Force
St. Louis 18, Missouri

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PREFACE

This manual is the second of the two part publication entitled "Map Accuracy Evaluation". Part I of the series consists of a thorough examination of the office method of evaluating cartographic materials for horizontal positioning information. There is no doubt that methods utilizing field surveys and checking symbolized or interpolated vertical data against surveyed points or photogrammetric methods where controlled aerial photography is used for evaluation are superior to other methods. However, the lack of photography and the inaccessibility of foreign areas preclude the use of such methods for many ACIC purposes.

The techniques previously used for office evaluations of vertical information varied with the analyst and evaluations were established either by estimate or a variation of the tangent-slope method. The method described in this manual considers the effect of scale, slope, contour interval, and horizontal displacement of contours on elevation error and provides a means for obtaining the average slope of terrain in the most rugged 10 percent of each area which is to be given a separate evaluation. The numerical value derived by the described method is considered the most realistic estimate of the Map Accuracy Standard (MAS) or 90 percent linear probability of the vertical accuracy which can be obtained by office methods.

1. Preliminary Evaluation

1.1. General Criteria. Many of the preliminary evaluation criteria discussed in Part I concern map elements applicable to vertical as well as horizontal evaluation. Much of the discussion regarding map scale, purpose, authority, significant dates, measurement units, and geodetic and cartographic history is also pertinent to the analysis and evaluation of vertical data and will not be repeated here. Of these elements, the geodetic and cartographic history of an area which applies to the type, location, and density of leveling surveys and methods by which the contours on the map were developed is of greatest value. When such information is available, an analyst is able to eliminate almost immediately those materials which do not fulfill the requirements of his assignment with respect to availability, adequacy, and precision of vertical information. Other preliminary evaluation criteria which are of particular significance to vertical accuracy evaluation include vertical datums, methods of relief portrayal and character of contour lines.

1.2. Specific Criteria.

1.2.1. Vertical Datums. The vertical datum is the level surface to which elevations are referred. Mean Sea Level (MSL) is the most commonly used vertical datum.

Sea Level is subject to unsystematic seasonal and yearly variations due to natural causes and therefore "Mean Sea Level" established at any specific location is not necessarily equal to

other mean sea level datums. However, the differences between the various Mean Sea Level Datums within continental limits have been determined not to exceed two meters and may be considered identical for most purposes. Mean Sea Level is obtained by averaging hourly heights of the sea observed on the open coast or in continuous waters with the observations taken over a considerable period of time to establish a valid statistical measurement. For this reason, the name of the place where mean sea level is established is usually included in the datum name (MSL of Amsterdam, Kronstadt, etc.).

Arbitrary vertical datums are sometimes employed as a basis for local leveling surveys. An arbitrary datum is an elevation assumed at some point and used as a base of reference for other elevations in the locality. With this type of datum, elevations are relative to each other and differences can be determined accurately without reference to mean sea level. Arbitrary datums occur most often in relatively undeveloped areas and the assumed datum may differ considerably from any MSL datum. The local surveys may later be tied and adjusted to a central leveling system and MSL datum. For example, there are certain areas of China where a correction must be applied to the Chinese maps to adjust vertical information to the approximate MSL datum.

The analyst should try to establish the vertical datum of each map selected for evaluation either by direct or comparative methods. Elevations are usually shown in meters or feet above or

below the vertical datum, however, other units are sometimes used, (e.g. - a sazhen used on old Russian maps equals 2.14 meters or 7 feet). When other units are used the exact conversion to meters or feet should be determined.

1.2.2. Relief Portrayal. The variation in height of the earth's surface is known as relief. Relief is shown on a map by one or a combination of the following forms: contours, color gradients, hachures, shading, form lines, special symbols, and spot elevations. Contours are the most common means of relief portrayal for areas with reliable vertical data. The hachure, shading, form line, and special symbol methods of relief portrayal present a visual expression of topography which is not suitable for precise determination of slope or elevation. Although spot elevations are limited to specific points they are usually the most accurate form of vertical data.

Contours. A contour is a line on which every point is an equal height above or below the vertical datum (reference surface). On maps, contours are assigned values designating the elevation above or below the reference surface.

The contour interval is the vertical distance between adjacent contours with different elevations. Factors considered in establishing the contour interval for a map include scale, slope, characteristics of the relief to be portrayed, and the purpose for which the map is intended.

There are a variety of types and accuracy classifications of

contours which should be recognized by the analyst. The types include the index contour, intermediate contour, supplementary or auxiliary contour, depression contour, and carrying contour. The accuracy classifications include the accurate or reliable contours and the approximate contours.

An index contour has a greater line weight than the others and serves as an aid to the user in locating contours with specific intervals. On most maps, every fifth contour beginning with the zero contour is a heavier line weight than the others and serves as an index contour.

Intermediate contours are those contours which appear between the index contours at the specified contour interval of the map. They may have an accurate or approximate value.

The supplementary or auxiliary contour is an additional contour interpolated between normal contours, usually at one half the contour interval, to portray significant features which would not be shown by the basic interval. They are symbolized by a dashed line and are added wherever necessary to clarify interpretation of the relief of the area. This type of contour may not be continuous.

A depression contour is any contour -- index, intermediate, supplementary, etc -- indicating a depression. The contour is shown as a closed loop with ticks on the downgrade side.

A carrying contour literally "carries" all the contours that converge in an area of vertical or near vertical slope.

An accurate or reliable contour is a solid contour line

that meets the accuracy specifications of the mapping authority.

An approximate contour is one that does not fulfill the standards of accuracy specified by the mapping authority for accurate or reliable contours. Approximate contours are portrayed as dashed lines and may be in the form of index contours, intermediate contours, and other types of contour lines.

The color gradient method of relief portrayal uses different colors or tones of the same color to indicate different zones of elevation. Each zone is bounded by contours and contours are often shown within the zones. The method of portrayal is usually used on small scale charts, e.g. the 1:1,000,000 scale International Maps of the World.

The hachure method of relief portrayal employs short lines drawn in the direction of steepest slope. The lines become heavier and closer together as the slope increases.

Shading produces an effect similar to the hachure methods. It uses variations of tone to color slopes in proportion to the degree of inclination. The steeper the slope the darker the shading.

Form lines are similar to contours but have neither numerical value nor definite interval. They are shown as full lines interspersed by broken lines of unequal length.

Spot elevations are numerical values generally assigned to the most prominent features in an area. The values may be determined barometrically, trigonometrically, or by spirit leveling, etc. Spot elevations which were determined by field surveys and identified on

the ground by a relatively permanent object are known as "bench marks". Spot elevations may appear alone or in conjunction with any or all of the previously described methods of relief portrayal.

1.2.3. Character of Contour Lines. Contour lines should portray accurately the character and the relative degree of relief dissection of the area shown on the map. A knowledge of the geomorphology of the area would enable the analyst to determine whether the contours actually conform to the existing terrain or whether they were generalized or animated by drawing arbitrary winding lines. The occurrence of an unusual number of parallel contours in a large area with extremely rugged terrain; parallel contours plotted uniformly and increasingly upward in areas of extremely rough relief; and greater prolongation of contours along tributary rather than main river valleys are all indications of generalized contours and mechanically selected detail. This type of contour drawing gives a distorted representation of slope for the region and contours are horizontally displaced. Should this go undetected by the analyst when evaluating such contours, the evaluation would be erroneous. If the horizontal displacement of contours is significantly greater than the displacement of planimetry, the amount should be determined and used in the final evaluation of vertical information. Other characteristics of contour lines that should be checked in evaluating maps for vertical information include:

1. Every contour is a closed curve even if the full contour is not shown on one map.

2. Contours never touch or cross except when cliffs and

overhanging cliffs are portrayed.

3. A contour never branches or splits.

4. Adjacent contours bear considerable resemblance to each other.

5. Most valley contours are "V" shaped while most ridge contours are "U" shaped.

6. When contours cross a stream, they always loop upstream.

2. Vertical Accuracy Evaluation of Topographic Maps

While the preliminary evaluation criteria are normally sufficient guides to the selection of specific source materials for vertical information, two important problems remain to be solved:

- (1) To establish area limits.
- (2) To determine the numerical value for each area.

The limit of the area for each separate evaluation is governed by the extent of area where significant accuracy differences can be obtained and the purpose for which the evaluations are being obtained. For example, certain special projects may require an accuracy evaluation for each nautical mile or larger sized square; other projects requiring an evaluation of vertical accuracy for the elevation of a specific point may necessitate a slope determination in the immediate vicinity of the point. However, for map evaluation in general, the size of the area will be governed by the scale of the source map, character of relief portrayal, and three distinct slope conditions of the terrain — 1) relatively level, 2) moderate or rolling hills, and 3) steep (steep hills, mountains, or karst regions). More specific guidance for defining evaluation areas is given in Part 3.

The numerical value represents the limits of vertical error for 90 percent of all identifiable features and is determined by a consideration of related mathematical elements.

2.1. Application of Error Theory. The Map Accuracy Standard (MAS) representing 90 percent probability is the linear precision index used to express vertical accuracies derived by the method described in this paper.¹ Final evaluations are stated in terms of \pm and rounded off according to the standard rounding off increments shown on the Vertical Map Accuracy Evaluation form.² The numerical value indicates that 90 percent of the elevations taken from the map and elevations interpolated from the contours on the map do not depart from the true elevations of those points by more than the indicated amount. It is necessary to assume that errors in elevations on or interpolated from contours follow a normal distribution.

2.2. Mathematical Elements. There are four inter-related mathematical elements which are of major significance in the evaluation of the vertical accuracy of topographic maps where relief is portrayed by contours -- 1) scale, 2) contour interval, 3) horizontal displacement, and 4) slope of terrain. A valid method of vertical accuracy evaluation must consider all of these elements and their relation to each other.

2.2.1. Scale. Since the scale of the map determines the amount of horizontal displacement and combined with slope determines

¹ ACIC Technical Report No. 96, "Principles of Error Theory and Cartographic Applications" Feb 1962.

² See attached sample form

the contour interval that can be portrayed, it has a vital relationship to the evaluation of vertical accuracy of maps. Scale is also a factor when determining the average slope by actual measurement of the number of contours crossing a line of specified distance on a map. Such measurements are used in conjunction with the simple and direct method for obtaining the tangent of the slope angle outlined in Chapter 3. While scale is an important factor in most vertical evaluation, it is not a necessary consideration when determining the highest elevation for a general area. In such an instance, the degree of assurance that all heights in the area are below the value indicated by the highest spot elevation is independent of scale.

2.2.2. Contour Interval. Several foreign countries, including the USSR¹, use the following formula to arrive at what is called the "normal" contour interval for use at a given scale:

$$N = b \tan \alpha$$

where N = the "normal" contour interval

b = the closest possible distance between lines on a map

$\tan \alpha$ = tangent of the angle of slope

The factor "b" is assumed to be 0.2 mm (equals .008 inch) on the map and therefore the ground distance is equal to $\frac{.2}{1000} S$ meters where S equals the scale denominator (e.g. 1:S).

¹ Sokolov, M. N. - The Contour Interval on Topographic Maps - Geodesy and Cartography No. 5-6 May-June 1960. English Translation Edition

The slope angle, α , is assumed to be 45° and therefore
 $\tan \alpha = 1$.

The formula results in the "normal" intervals shown in the following table. The range of intervals actually used for the scales is also shown.

| <u>Map Scale</u> | <u>Contour interval in meters</u> | |
|------------------|-----------------------------------|---------------------|
| | <u>Normal</u> | <u>Used on Maps</u> |
| 1:5,000 | 1 | .25 - 10 |
| 1:10,000 | 2 | .25 - 10 |
| 1:25,000 | 5 | 1 - 10 |
| 1:50,000 | 10 | 10 - 20 |
| 1:100,000 | 20 | 10 - 40 |

The portrayal of a 45° slope would be practically impossible with the "normal" interval because it would require drawing 125 lines per inch. However, since such a steep slope is rather rare in nature, the normal interval has been established by experience as a satisfactory value to use for moderate terrain. In steep or mountainous regions, the normal interval is doubled.

The preceding formula can be useful in checking whether or not the compiler used good judgement in selecting the interval portrayed on the map. When using it for this purpose, the average slope of the area under investigation should be substituted for the 45° slope and a more reasonable minimum spacing between adjacent contours (e.g. 1mm or 25 lines per inch) should be substituted for (factor "b") the closest possible distance between lines on a map.

2.2.3. Horizontal Displacement of Contours. The U.S. National Map Accuracy Standards¹ contain the following statement with regard to vertical accuracy:

"Vertical accuracy as applied to contour maps on all publication scales, shall be such that not more than 10 percent of the elevations tested shall be in error more than one-half the contour interval. In checking elevations taken from the map the apparent vertical error may be decreased by assuming a horizontal displacement within the permissible horizontal error for a map of that scale".

Most foreign countries and the North Atlantic Treaty Organization (NATO) have comparable standards for first class maps. Although they usually express the allowable error in terms of the statistical measure of standard deviation rather than 90 percent probability they all contain a statement or formula providing an additional allowance for horizontal displacement.² The effect of the allowable horizontal displacement on the vertical accuracy of maps varies with slope and is strikingly illustrated by the diagrams included as Appendix B and C.

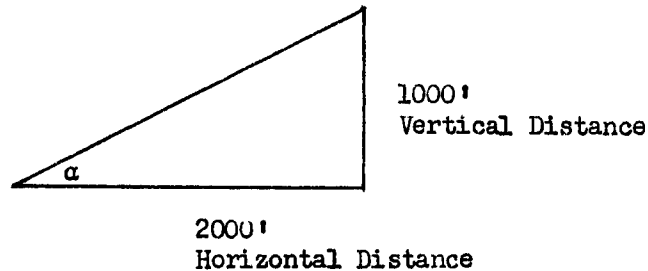
¹ Thompson, Morris M. and Davey, Charles H. (USGS) "Vertical Accuracy of Topographic Maps" Surveying and Mapping - Vol. XIII - No. 1 - Jan - Mar 1953.

² Thompson, Morris, M. (USGS) - "A Current View of National Map Accuracy Standards" Surveying and Mapping - Vol. XX No. 4 - Dec. 1960.

The effect of the allowable horizontal displacement clause can result in vertical accuracies considerably greater than half the contour interval on even the highest quality maps. It has the effect of increasing the vertical tolerance as the ground slope increases according to the mathematical expression:

$$h \tan \alpha$$

where "h" equals the allowable horizontal shift and $\tan \alpha$ = the tangent of the angle of slope. For example on a USGS 1:24,000 scale map which meets National Map Accuracy Standards $h = .02$ in. or 40 ft. in ground distance. If the slope in an area of the map showed a 1000 ft. rise in a 2000 ft. distance:



$$\tan \alpha = \frac{1000}{2000} = 0.5$$

Then the allowable error due to the horizontal shift is:

$$h \tan \alpha = 40 (.5) = 20 \text{ ft.}$$

The vertical error of 20 ft. is in addition to the one half contour interval vertical tolerance. If the contour interval "C" shown on the map just described is 20 ft., the acceptable vertical accuracy of 90 percent of the points in the area of the above indicated slope is:

$$\frac{C}{2} + 20 \text{ or } \frac{20}{2} + 20 = 30 \text{ Ft.}$$

This is equivalent to one and one half times the contour interval and yet it still meets vertical accuracy standards for a first class map.

If the compiler had used a 50 ft. contour interval $\frac{C}{2}$ would equal 25 ft. and the allowable tolerance to meet map accuracy standards in the preceding example is $(25 + 20)$ 45 ft. or 0.9 of the contour interval. A 50 ft. interval, which would be a more reasonable¹ interval to use for a slope as steep as this, would require 20 lines per inch to portray a 1000' rise in a distance of 2000 ft. at the scale indicated.

The examples just cited illustrate the effect of horizontal displacement on vertical error with the allowable tolerance (.02") for first class maps. The vertical error will be proportionately greater where the horizontal displacement of contours is greater. Class "B" maps, for example, which have been evaluated as having .04" horizontal displacement (90 percent probability) will have double the vertical error for the same slopes as shown in the examples for first class maps.

The horizontal displacement of contours under normal conditions is the same as the CMAS of the planimetry which is established as a result of the evaluation of the horizontal accuracy of the map. The only time a larger estimated value for horizontal displacement should be used is when contours appear to be overly generalized or

¹ Griffith, Shirley V. - (USC&GS) Contouring Problems on General-Purpose Maps Surveying and Mapping. Vol. XII No. 4 - Oct-Dec 1952.

compiled from secondary source independent of the planimetry, or when other errors (as described in the Section on Preliminary Evaluation) exist.

2.2.4. Slope. The term "slope" is used in this paper to mean the degree of deviation from the horizontal. It may be measured in degrees of the acute angle formed by the intersection of the inclined surface and the horizontal surface, or in terms of the percent of vertical rise per unit of horizontal distance. The latter is the same as the tangent of the angle of slope.

The relation of slope to scale, contour interval, and horizontal displacement has already been discussed. For the purpose of the evaluation of vertical accuracy, the scale and contour interval are given or can easily be determined. The horizontal displacement of contours is normally the same as the CMAS of the planimetry. In exceptional cases it is estimated on the basis of factors described under Preliminary Evaluation. The determination of slope over an area on a map or series of maps requires a certain amount of cartographic judgment. When the slope at a single point on a map is to be determined, it can usually be obtained accurately from the adjacent contours.

One method of determining average slope for an area of a map is described in a paper by C. K. Wentworth.¹ The method involves

¹ Wentworth, C. K. - A Simplified Method of Determining the Average Slope of Land Surfaces. American Journal of Science Series 5, Volume 20 (New Haven, Conn., 1930)

counting contour crossings along a number of arbitrary grid lines laid over the area and establishing the average number of contour crossings per mile. The data, when applied in a formula using the contour interval and scale, results in the tangent of the angle of slope.

Since the numerical values of the vertical accuracy evaluation for Air Force use is usually desired in terms of 90 percent probability rather than standard deviation, it is more convenient to determine the vertical error directly in terms of 90 percent probability. By computing the error caused by the slope of the most rugged 10 percent of the terrain a value within the 90 percent probability is assured. It requires no greater cartographic judgment to select the area of a map that is typical of the most rugged 10 percent of the area being evaluated than to determine the average slope. Within the most rugged 10 percent of the area, the tangent of the slope angle (α) can be determined directly by the following formula:

$$\tan \alpha = \frac{a \times C}{S/12} \text{ or } \frac{12Ca}{S}$$

where a = the average Nr. of contours per inch

C = the contour interval in feet

S = the scale denominator (e.g. 1:S)

$S/12$ = Ground Distance in feet per inch of map distance.

Since the scale (S) and contour interval (C) are known, the method of determining " a " is the only place where cartographic judgment is involved. A simple and quick method of arriving at

this value is to count the number of contour lines per inch crossing a minimum of 10 separate lines perpendicular to contours typical of the most rugged 10 percent of the terrain to be evaluated and obtain the mean value.

3. Procedure for Vertical Accuracy Evaluation of Topographic Maps

3.1. Tangent-Slope Formula. On the basis of the elements just discussed, the following formula is proposed for making a logical estimation of the vertical accuracy of maps:

$$\text{MAS} = 0.5 C + \text{CMAS} \tan \alpha$$

where: MAS = Map Accuracy Standard or the accuracy of vertical map information at 90 percent linear probability.

CMAS = Circular Map Accuracy Standard or the accuracy of horizontal map information at 90 percent circular probability.

C = Contour Interval

$$\tan \alpha = \text{Tangent of Slope angle} = \frac{12Ca}{S}$$

where: S = Scale Denominator (e.g. 1:S)

a = Average Nr. of Contours per inch in most rugged 10 percent of area selected for evaluation.

The formula encompasses all of the mathematical elements previously described in their proper relationship as they affect vertical accuracy. Testing has shown that fairly consistent results are obtained with the formula when evaluations are performed independently by different analysts.

In most cases, the factor "a" used to compute the tangent of the slope is the only variable which requires experienced cartographic judgment for derivation. Normally, the CMAS factor is the same as the value already derived for the horizontal accuracy of planimetry. The only time a larger estimated value will be used is

where the contours on the map have significantly greater horizontal displacement than the planimetry. A CMAS value of less than 0.02 inch at map scale is rarely found even for maps of the highest quality. The factor S for Scale Denominator and C for Contour Interval are usually given or can easily be established.

The factor $0.5C$ (one half contour interval) represents the optimum value for vertical accuracy of topographic contour maps in fairly level terrain. If the terrain is level, the slope angle is zero and the second part of the formula ($CMAS \tan \alpha$) is equal to zero. The vertical accuracy of 90 percent of elevations derived from contour maps in fairly level terrain is very close to one half contour interval. In more rugged terrain an additional error proportionate to the horizontal displacement of contours times the tangent of the slope angle is added to the half contour interval. Since the method of computing the tangent of the angle of slope described in this paper involves direct measurement of contours per inch and considers both scale and contour interval, it is self-compensating and can be applied equally well to large and small scale maps. For example, small scale maps usually have large CMAS factors and small $\tan \alpha$ factors and the resulting error allowance usually is a reasonable proportion of the contour interval;

3.2. Limits of Area for a Single Evaluation. The first step in the procedure for the evaluation of vertical accuracy is determining the extent of area to be given a single evaluation. This

can be a portion of a map, a complete map, or a series of maps. The following criteria should be used as a guide for making this determination.

For maps 1:250,000 scale or larger, a complete map sheet will be the minimum area to be given a separate evaluation - - except where there is a change in contour interval within a sheet or a distinct change in the character or type of relief portrayed on a sheet. e.g. A change from a 20 to 40 meter contour interval for rugged terrain above 1000 meter line on a 1:100,000 scale map; or a distinct portion of a large scale map showing approximate contours or form lines while remainder of map shows accurate contours.

For maps smaller than 1:250,000 scale, divide the sheet into not more than three distinct terrain regions, as necessary for any one contour interval -1.) relatively level, 2.) moderate or (rolling hills) 3.) steep (steep hills, mountains, or karst regions). Where varying contour intervals are used on the same map only one accuracy evaluation will normally be given for each contour interval used.

3.3. Determination of the Tangent of Slope Angle. The determination of the Tangent of the Slope angle involves the three variables shown in the formula:

$$\tan \alpha = \frac{12Ca}{S}$$

Since the contour interval (C) and the scale (S) are usually given, the only factor to be obtained from actual measurement on the map is "a". The following method is proposed for determining "a":

Count the number of lines per inch which cross a minimum of 10 separate lines perpendicular to contours typical of the terrain in the most rugged 10 percent of the area and obtain a mean value.

The tangent of the slope angle can then be computed. In using the formula, it is important to remember to convert the contour interval to feet since the factor S/12 represents the ground distance of one inch of map distance in feet.

Example 1

$$\text{Map Scale} = \frac{1}{S} = 1:24,000$$

$$\text{Contour Interval} = 10 \text{ ft.}$$

$$a = 40$$

$$\tan \alpha = \frac{12 \times 40 \times 10}{24,000} =$$

$$\frac{4,800}{24,000} = 0.200$$

Example 2

$$\text{Map Scale} = \frac{1}{S} = 1:1,000,000$$

$$\text{Contour Interval} = 50 \text{ meters} = 164 \text{ ft.}$$

$$a = 25$$

$$\tan \alpha = \frac{12 \times 25 \times 164}{1,000,000} =$$

$$\frac{49,200}{1,000,000} = 0.049$$

3.4. Determination of Vertical Accuracy.¹ When the tangent of the slope angle has been computed, the final vertical accuracy evaluation can be derived by applying the formula:

$$\text{MAS} = 0.5C + \text{CMAS} \tan \alpha$$

For Example 1

$$\begin{aligned} \text{CMAS} &= 0.02 \text{ in. Map Distance} \\ &= 40 \text{ ft. Ground Distance} \end{aligned}$$

$$\tan \alpha = 0.200$$

$$\begin{aligned} \text{MAS} &= 5 + 40 (.2) \\ &= 5 + 8 = \pm 13 \text{ ft.} \\ \text{rounded off} &= \pm 10 \text{ ft.} \end{aligned}$$

For Example 2

$$\begin{aligned} \text{CMAS} &= 0.04 \text{ in. Map Distance} \\ &= 3333 \text{ ft. Ground Distance} \end{aligned}$$

$$\tan \alpha = 0.049$$

$$\begin{aligned} \text{MAS} &= 82 + 3333 (.049) \\ &= 82 + 163 = \pm 245 \text{ ft.} \\ \text{rounded off} &= \pm 250 \text{ ft.} \end{aligned}$$

¹ See attached sample form

4. Accuracy of Spot Elevations

Spot elevations serve as a basis for drawing contours by interpolation or photogrammetric methods. For optimum relief portrayal, spot elevations should be located in sufficient quantity to establish characteristic heights, cols, passes, lowest points of depressions, slopes of low lands, etc. The most desirable number of spot elevations depends on the degree of dissection of the relief. Areas of rough or rugged terrain require more spot elevations than areas of low or moderate relief.

The density and distribution of spot elevations is often an indication of the vertical accuracy. An area with a good pattern of spot elevations is usually accurate, while an area with very few spot elevations portrays the relief with little vertical control. The evaluation of spot elevations should be based on the results of a direct and comparative analysis to determine the method by which they were derived and the reliability of the producer. The analysis should take into consideration the units of measurement, vertical survey methods, horizontal control, and comparison with available textual and cartographic materials.

4.1. Units of Measurement. The unit of measurement for the spot elevations is usually indicated in the margin of the sheet. If they are shown in any unit other than feet the correct conversion factor must be determined, (meters to feet, sazhen to feet etc). Sometimes different units are used on the same sheet. For example, the elevations may be given in feet while soundings are

given in fathoms.

4.2. Vertical Survey Methods. Spot elevations are usually established by differential leveling, trigonometric leveling, or barometric leveling.

Differential Leveling is a method of determining elevations of points with respect to each other by means of an instrument using a spirit level to establish the horizontal line of sight. It is the most accurate of the vertical survey methods and is the only method used to determine first order level nets. Points established by this method are usually fixed on a more or less permanent object or monument and shown on the map as bench marks. Second and third order points may also be determined by this method. Elevations determined by differential leveling can be considered accurate within ± 10 feet.

Trigonometric Leveling is the determination of differences of elevations by means of observed vertical angles combined with the lengths of lines. This method can be used to establish second and third order nets, but generally it is used for fourth order nets. Elevations determined by trigonometric leveling can be considered accurate within the standard rounding off increments used by ACIC for vertical data.¹

Barometric Leveling is a method of determining differences of elevation from differences of atmospheric pressure observed with a

¹ See attached sample form

barometer. Elevations obtained in this way are shown only as spot elevations on a map and are not recoverable in the field. The accuracies of barometric elevations may range from a few feet to + 100 feet or more.

4.3. Horizontal Control. Horizontal control is the framework to which all planimetric detail and relief information is held. The vertical data (spot elevations etc.) on a map can only be as good as the horizontal control. It is not possible for a map to exhibit good vertical control and poor horizontal control; although, good horizontal control on a map does not necessarily indicate good vertical control.

4.4. Comparative Methods

4.4.1. Textual Materials. If textual material (journals, catalogues, and field books including diagrams of precision and distribution of vertical control networks etc.) are available, the analyst can make a direct comparison of the listed values with the values shown on the map. If descriptions of points are available that are not shown on the map, they can be plotted and a comparison made between the elevation values given and values interpolated from contours on the map. By analyzing the differences obtained and knowing the precision of the surveys the analyst can arrive at a numerical evaluation of the accuracy of spot elevations on the map.

4.4.2. Other Cartographic Materials. A comparison of spot elevations on the map being evaluated with heights of the same features on other cartographic source materials often reveals a

variety of discrepancies. The analysis of the cause of these discrepancies and the evaluations based on the results of such comparisons should consider the following factors.

Policy governing rounding off varies among the different authorities. Rounding off may be to a whole unit; to the nearest tens or hundreds; or by the Gauss rule.

In resolving differences caused by varying methods of height determination it will be necessary to review the method and techniques used by each of the authorities and choose the value determined by the more precise method as nearer the true value. In the event this information is not available or the authorities used equally comparable methods, the analyst must rely on the relative dependability of the authorities.

Often in overlapping areas of maps compiled by two different governments discrepancies which approximate a constant occur between identical spot elevations. This can usually be rectified by determining the vertical datum on which each system is based.

4.5. Estimated Accuracy of Spot Elevations. If, after considering the evaluation criteria and techniques just discussed, the resulting information is insufficient for making a precise evaluation, the accuracy of spot elevations will be estimated. Normally spot elevations are more accurate than the contours so an arbitrary estimate of their accuracy at 90 percent probability may be obtained by using the optimum MAS of contour accuracy or one half of the con-

tour interval. This value should be rounded-off in accordance with the standard rounding-off increments.

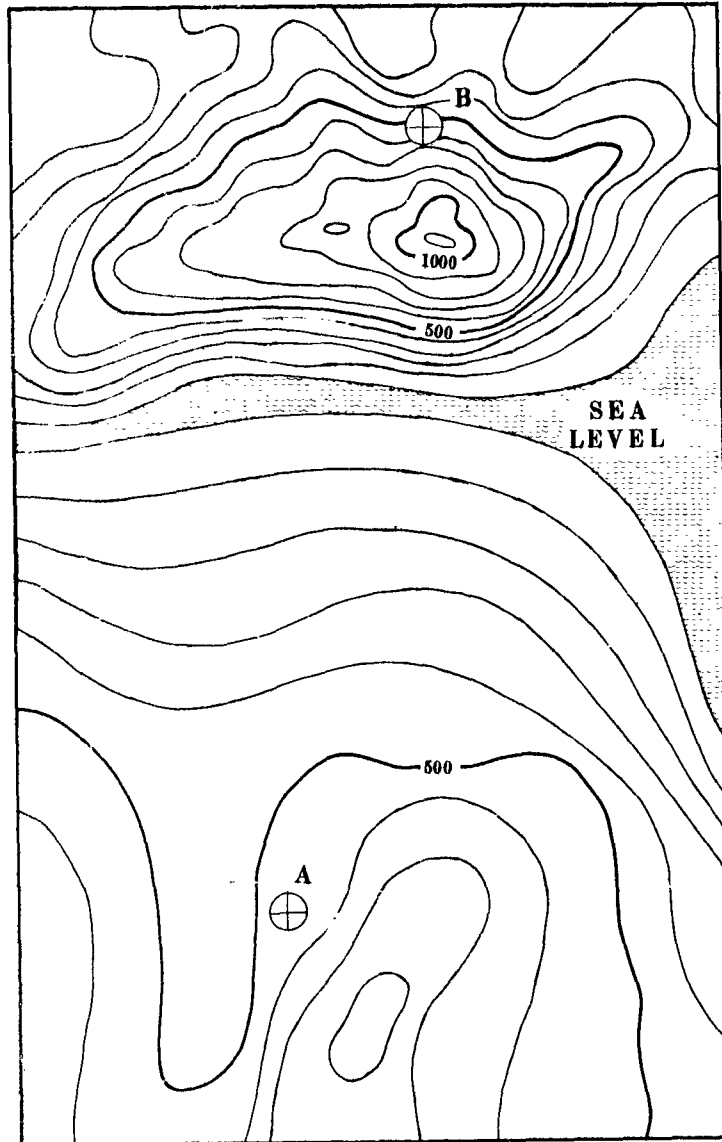
TABLE OF SELECTED CONTOUR INTERVALS IN FEET

| C.I. In Meters | 0.5 Contour Interval | | One Contour Interval | |
|-------------------|----------------------|-----------|----------------------|-----------|
| | Precise | Rounded * | Precise | Rounded * |
| 5 | 8.2 | 10 | 16.4 | 20 |
| 10 | 16.4 | 20 | 32.8 | 30 |
| 15 | 24.6 | 20 | 49.2 | 50 |
| 20 | 32.8 | 30 | 65.6 | 70 |
| 25 | 41.0 | 40 | 82.0 | 80 |
| 30 | 49.2 | 50 | 98.4 | 100 |
| 40 | 65.6 | 70 | 131.2 | 130 |
| 50 | 82.0 | 80 | 164.0 | 160 |
| 75 | 123.0 | 120 | 246.1 | 250 |
| 80 | 131.2 | 130 | 262.4 | 250 |
| 100 | 164.0 | 160 | 328.1 | 350 |
| 150 | 246.1 | 250 | 492.1 | 500 |
| 200 | 328.1 | 350 | 656.2 | 700 |
| 250 | 410.1 | 400 | 820.2 | 800 |
| 300 | 492.1 | 500 | 984.2 | 1000 |
| 500 | 820.2 | 800 | 1640.4 | 1600 |
| 1000 | 1640.4 | 1600 | 3280.8 | 3500 |

* Standard Rounding-Off increments used as shown on attached sample form.

Appendix B

EFFECT OF SLOPE ON ELEVATION ERROR



*The radius of circle around each point represents the degree of uncertainty in horizontal position or CMAS
CMAS = Circular map accuracy standard (90% Probability)*

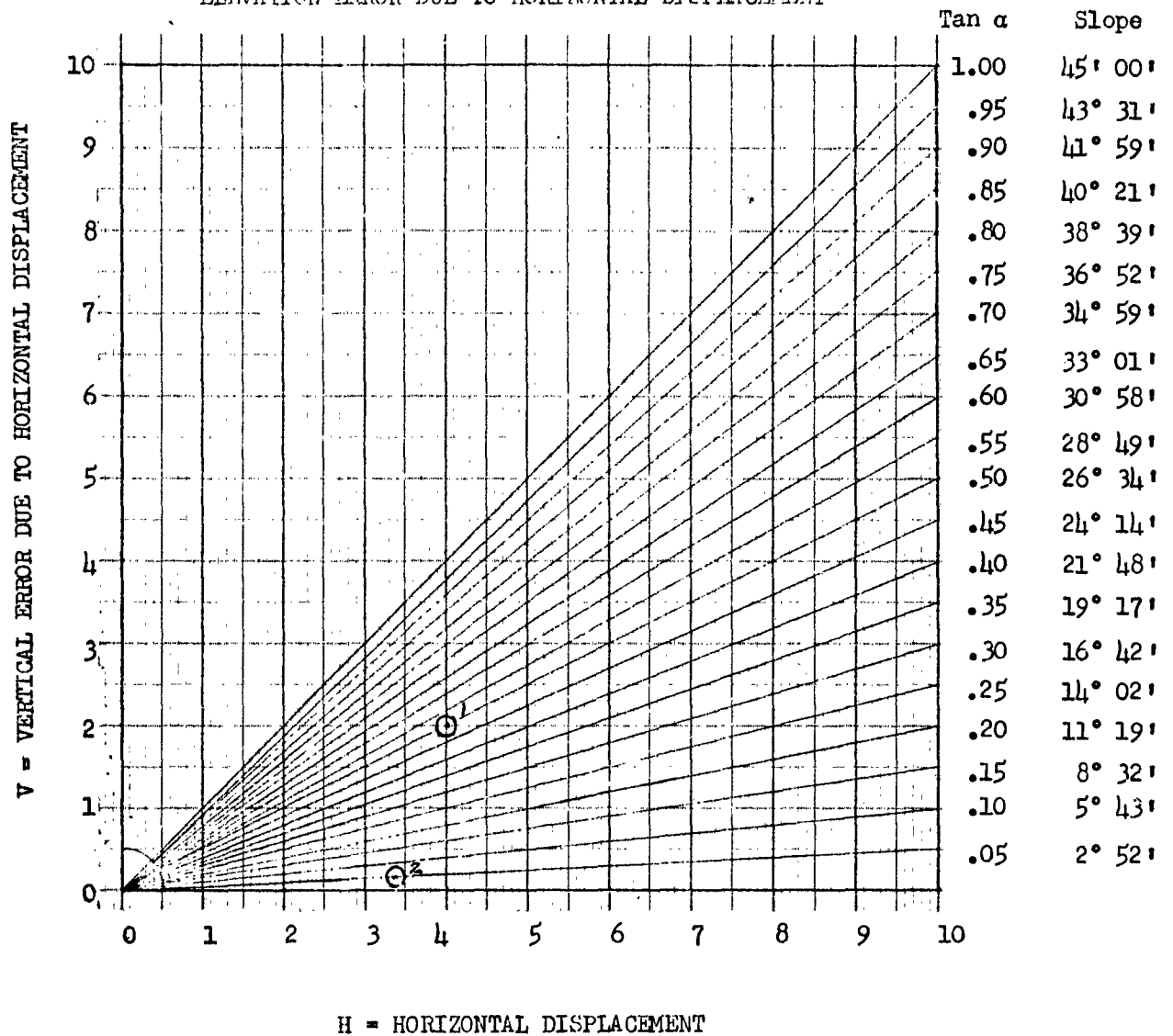
Elevations of points interpolated from contours

POINT A = 550 feet ± 50 feet

POINT B = 550 feet ± 150 feet

Appendix C

ELEVATION ERROR DUE TO HORIZONTAL DISPLACEMENT



Note: For greater values of H and V multiply each by powers of ten, (e.g. 10^1 , 10^2 , 10^3 10^n).

Example 1

Map Scale = 1:24,000

H = .02 in. (M.D.) = 40 ft (G.D.)

tan α = .50

V = 20 ft.

Example 2

Map Scale = 1:1,000,000

H = .04 in (M.D.) = 3333 ft (G.D.)

tan α = .05

V = 170 ft.

VERTICAL MAP ACCURACY EVALUATION

LIB. CALL NO. _____ SHEET NO. _____

AREA: a. ☐ entire sheet
b. ☐ portion of sheet (define): _____

VERTICAL DATUM: _____

S = Scale Denominator (e.g. 1:S) =

C = Contour Interval = meters = feet

a = Average No. of Contours per inch =

Tan α = Tangent of Slope Angle* = $\frac{12 Ca}{S}$ =

COMPUTATIONS:

MAS = Map Accuracy Standard or the accuracy of vertical map information at 90 percent linear probability.

MAS = $0.5C + CMAS ** \tan \alpha$

MAS = $0.5C + (CMAS **) \times (\tan \alpha)$ = feet

MAS = + \times = feet

ROUNDING-OFF INCREMENTS

| MAS (ft.) | Increment | 1000-2000 | 100 |
|-----------|-----------|-----------|------|
| 0-200 | 10 | 2000-5000 | 500 |
| 200-1000 | 50 | Over 5000 | 1000 |

FINAL MAS = \pm feet

*In most rugged 10 percent of area

**CMAS = Circular Map Accuracy Standard or the accuracy of horizontal map information at 90 percent circular probability. Normally, the value obtained for horizontal accuracy evaluation of planimetry (90 percent probability) will be used. In cases where contours are evaluated as having significantly greater horizontal displacement, the estimated value of horizontal displacement of contours should be used.